Water-Quality Principles
QW1022–TEL

Lesson 5, Part 1—Field-Measured Properties and Major Ions: Field Measurements

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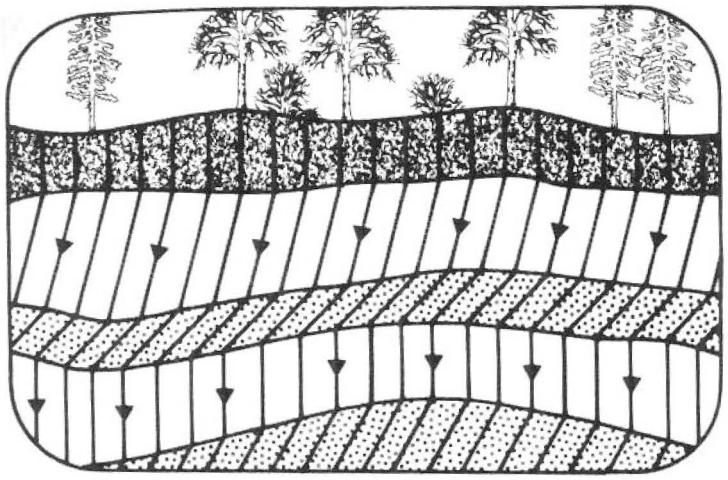

5.1 Flow Nets by Graphical Construction

We have seen in Chapter 2 that a groundwater flow system can be represented by a three-dimensional set of equipotential surfaces and a corresponding set of orthogonal flowlines. If a meaningful two-dimensional cross section can be chosen through the three-dimensional system, the set of equipotential lines and flowlines so exposed constitutes a *flow net*. The construction of flow nets is one of the most powerful analytical tools for the analysis of groundwater flow.

In Section 2.11 and Figure 2.25, we saw that a flow net can be viewed as the solution of a two-dimensional, steady-state, boundary-value problem. The solution requires knowledge of the region of flow, the boundary conditions on the boundaries of the region, and the spatial distribution of hydraulic conductivity within the region. In Appendix III, an analytical mathematical method of solution is presented. In this section, we will learn that flow nets can also be constructed graphically, without recourse to the sophisticated mathematics.

Homogeneous, Isotropic Systems

Let us first consider a region of flow that is homogeneous, isotropic, and fully saturated. For steady-state flow in such a region, three types of boundaries can exist: (1) impermeable boundaries, (2) constant-head boundaries, and (3) water-table boundaries. First, let us consider flow in the vicinity of an impermeable boundary [Figure 5.1(a)]. Since there can be no flow across the boundary, the flowlines adjacent to the boundary must be parallel to it, and the equipotential lines must meet the boundary at right angles. By invoking Darcy's law and setting the specific discharge across the boundary equal to zero, we are led to the mathematical statement of the boundary condition. For boundaries that parallel the axes in an xz plane:

$$\frac{\partial h}{\partial x} = 0 \quad \text{or} \quad \frac{\partial h}{\partial z} = 0 \tag{5.1}$$

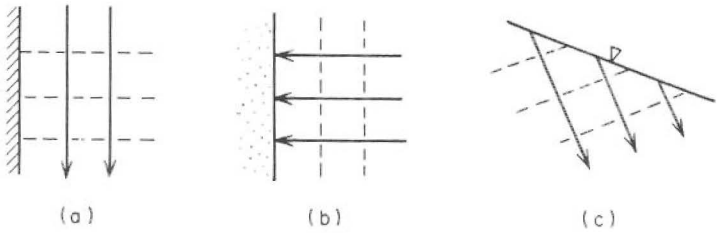

Figure 5.1 Groundwater flow in the vicinity of (a) an impermeable boundary, (b) a constant-head boundary, and (c) a water-table boundary.

In effect, any flowline in a flow net constitutes an imaginary impermeable boundary, in that there is no flow *across* a flowline. In flow-net construction, it is often desirable to reduce the size of the region of flow by considering only those portions of the region on one side or the other of some line of symmetry. If it is clear that the line of symmetry is also a flowline, the boundary condition to be imposed on the symmetry boundary is that of Eq. (5.1).

A boundary on which the hydraulic head is constant [Figure 5.1(b)] is an equipotential line. Flowlines must meet the boundary at right angles, and adjacent equipotential lines must be parallel to the boundary. The mathematical condition is

$$h = c \tag{5.2}$$

On the water table, the pressure head, ψ, equals zero, and the simple head relationship, $h = \psi + z$, yields

$$h = z \tag{5.3}$$

for the boundary condition. As shown in Figure 5.1(c), for a recharge case the water table is neither a flowline nor an equipotential line. It is simply a line of variable but known h.

If we know the hydraulic conductivity K for the material in a homogenous, isotropic region of flow, it is possible to calculate the discharge through the system from a flow net. Figure 5.2 is a completed flow net for the simple case first presented in Figure 2.25(a). The area between two adjacent flowlines is known as a *streamtube* or *flowtube*. If the flowlines are equally spaced, the discharge through each streamtube is the same. Consider the flow through the region $ABCD$ in Figure 5.2. If the distances AB and BC are ds and dm, respectively, and if the hydraulic-head drop between AD and BC is dh, the discharge across this region through a cross-sectional area of unit depth perpendicular to the page is

$$dQ = K\frac{dh}{ds}\,dm \tag{5.4}$$

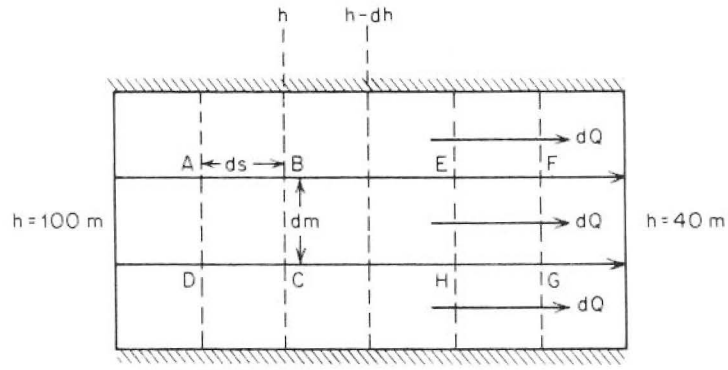

Figure 5.2 Quantitative flow net for a very simple flow system.

Under steady-state conditions, the discharge across any plane of unit depth (say, at AD, EH, or FG) within the streamtube must also be dQ. In other words, the discharge through any part of a streamtube can be calculated from a consideration of the flow in just one element of it.

If we arbitrarily decide to construct the flow net in squares, with $ds = dm$, then Eq. (5.4) becomes

$$dQ = K\,dh \tag{5.5}$$

For a system with m streamtubes, the total discharge is

$$Q = mK\,dh \tag{5.6}$$

If the total head drop across the region of flow is H and there are n divisions of head in the flow net ($H = n\,dh$), then

$$Q = \frac{mKH}{n} \tag{5.7}$$

For Figure 5.2, $m = 3$, $n = 6$, $H = 60$ m, and from Eq. (5.7), $Q = 30K$. For $K = 10^{-4}$ m/s, $Q = 3 \times 10^{-3}$ m^3/s (per meter of section perpendicular to the flow net).

Equation (5.7) must be used with care. It is applicable only to simple flow systems with one recharge boundary and one discharge boundary. For more complicated systems, it is best to simply calculate dQ for one streamtube and multiply by the number of streamtubes to get Q.

Figure 5.3 is a flow net that displays the seepage beneath a dam through a foundation rock bounded at depth by an impermeable boundary. It can be used to make three additional points about flow-net construction.

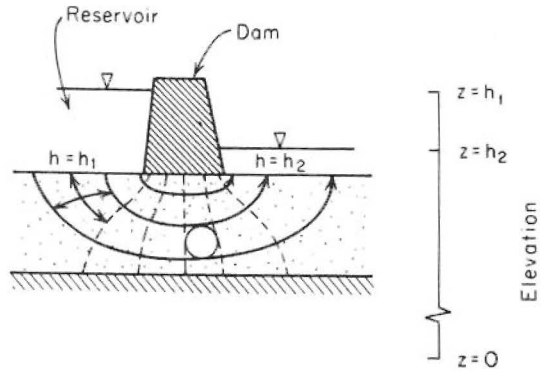

Figure 5.3 Seepage beneath a dam through homogeneous, isotropic foundation rocks.

1. The "squares" in all but the simplest flow nets are actually "curvilinear" squares; that is, they have equal central dimensions; or viewed another way, they enclose a circle that is tangent to all four bounding lines.
2. It is not necessary that flow nets have finite boundaries on all sides; regions of flow that extend to infinity in one or more directions, like the horizontally infinite layer in Figure 5.3, are tractable.
3. A flow net can be constructed with a "partial" streamtube on the edge.

For the flow net shown in Figure 5.3, $m = 3\frac{1}{2}$. If $H = 100$ m and $K = 10^{-4}$ m/s, then, since $n = 6$, we have $Q = 5.8 \times 10^{-3}$ m^3/s (per meter section perpendicular to the flow net).

In homogeneous, isotropic media, the distribution of hydraulic head depends only on the configuration of the boundary conditions. The qualitative nature of the flow net is independent of the hydraulic conductivity of the media. The hydraulic conductivity comes into play only when quantitative discharge calculations are made. It is also worth noting that flow nets are dimensionless. The flow nets of Figures 5.2 and 5.3 are equally valid whether the regions of flow are considered to be a few meters square or thousands of meters square.

The sketching of flow nets is something of an art. One usually pursues the task on a trial-and-error basis. Some hydrologists become extremely talented at arriving at acceptable flow nets quickly. For others, it is a source of continuing frustration. For a flow net in homogeneous, isotropic media, the rules of graphical construction are deceptively simple. We can summarize them as follows: (1) flowlines and equipotential lines must intersect at right angles throughout the system; (2) equipotential lines must meet impermeable boundaries at right angles; (3) equipotential lines must parallel constant-head boundaries; and (4) if the flow net is drawn such that squares are created in one portion of the field, then, with the

possible exception of partial flow tubes at the edge, squares must exist throughout the entire field.

Heterogeneous Systems and the Tangent Law

When groundwater flowlines cross a geologic boundary between two formations with different values of hydraulic conductivity, they refract, much as light does when it passes from one medium to another. However, in contradistinction to Snell's law, which is a sine law, groundwater refraction obeys a tangent law.

Consider the streamtube shown in Figure 5.4. Flow proceeds from a medium

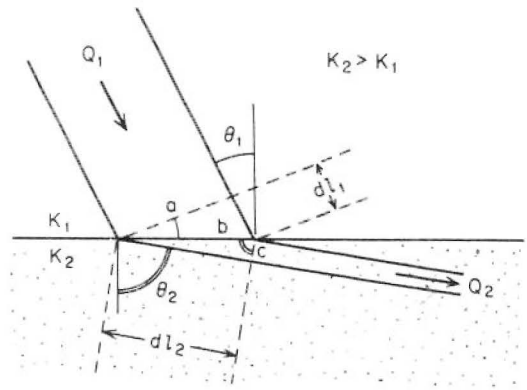

Figure 5.4 Refraction of flowlines at a geologic boundary.

with hydraulic conductivity K_1 to a medium with hydraulic conductivity K_2, where $K_2 > K_1$. The streamtube has a unit depth perpendicular to the page, and the angles and distances are as indicated on the figure. For steady flow, the inflow Q_1 must equal the outflow Q_2; or, from Darcy's law,

$$K_1 a \frac{dh_1}{dl_1} = K_2 c \frac{dh_2}{dl_2} \tag{5.8}$$

where dh_1 is the head drop across the distance dl_1 and dh_2 is the head drop across the distance dl_2. In that dl_1 and dl_2 bound the same two equipotential lines, it is clear that $dh_1 = dh_2$; and from geometrical considerations, $a = b \cos \theta_1$ and $c = b \cos \theta_2$. Noting that $b/dl_1 = 1/\sin \theta_1$ and $b/dl_2 = 1/\sin \theta_2$, Eq. (5.8) becomes

$$K_1 \frac{\cos \theta_1}{\sin \theta_1} = K_2 \frac{\cos \theta_2}{\sin \theta_2} \tag{5.9}$$

or

$$\frac{K_1}{K_2} = \frac{\tan \theta_1}{\tan \theta_2} \tag{5.10}$$