Water-Quality Principles
QW1022–TEL

Lesson 4—Governing Principles of Aqueous Systems

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U.S. Department of the Interior
U.S. Geological Survey

2.1 Darcy's Law

The birth of groundwater hydrology as a quantitative science can be traced to the year 1856. It was in that year that a French hydraulic engineer named Henry Darcy published his report on the water supply of the city of Dijon, France. In the report Darcy described a laboratory experiment that he had carried out to analyze the flow of water through sands. The results of his experiment can be generalized into the empirical law that now bears his name.

Consider an experimental apparatus like that shown in Figure 2.1. A circular cylinder of cross section A is filled with sand, stoppered at each end, and outfitted with inflow and outflow tubes and a pair of manometers. Water is introduced into the cylinder and allowed to flow through it until such time as all the pores are filled with water and the inflow rate Q is equal to the outflow rate. If we set an arbitrary datum at elevation $z = 0$, the elevations of the manometer intakes are

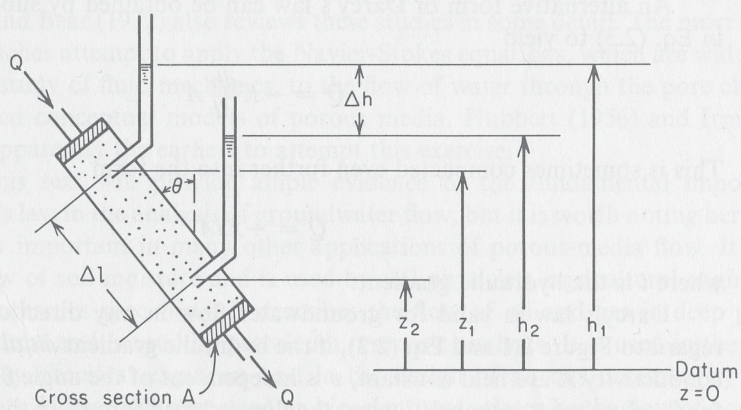

Figure 2.1 Experimental apparatus for the illustration of Darcy's law.

z_1 and z_2 and the elevations of the fluid levels are h_1 and h_2. The distance between the manometer intakes is Δl.

We will define v, the *specific discharge* through the cylinder, as

$$v = \frac{Q}{A} \tag{2.1}$$

If the dimensions of Q are $[L^3/T]$ and those of A are $[L^2]$, v has the dimensions of a velocity $[L/T]$.

The experiments carried out by Darcy showed that v is directly proportional to $h_1 - h_2$ when Δl is held constant, and inversely proportional to Δl when $h_1 - h_2$ is held constant. If we define $\Delta h = h_2 - h_1$ (an arbitrary sign convention that will stand us in good stead in later developments), we have $v \propto -\Delta h$ and $v \propto 1/\Delta l$. *Darcy's law* can now be written as

$$v = -K\frac{\Delta h}{\Delta l} \tag{2.2}$$

or, in differential form,

$$v = -K\frac{dh}{dl} \tag{2.3}$$

In Eq. (2.3), h is called the *hydraulic head* and dh/dl is the *hydraulic gradient*. K is a constant of proportionality. It must be a property of the soil in the cylinder, for were we to hold the hydraulic gradient constant, the specific discharge would surely be larger for some soils than for others. In other words, if dh/dl is held constant, $v \propto K$. The parameter K is known as the *hydraulic conductivity*. It has high values for sand and gravel and low values for clay and most rocks. Since Δh and Δl both have units of length $[L]$, a quick dimensional analysis of Eq. (2.2) shows that K has the dimensions of a velocity $[L/T]$. In Section 2.3, we will show that K is a function not only of the media, but also of the fluid flowing through it.

An alternative form of Darcy's law can be obtained by substituting Eq. (2.1) in Eq. (2.3) to yield

$$Q = -K\frac{dh}{dl}A \tag{2.4}$$

This is sometimes compacted even further into the form

$$Q = -KiA \tag{2.5}$$

where i is the hydraulic gradient.

Darcy's law is valid for groundwater flow in any direction in space. With regard to Figure 2.1 and Eq. (2.3), if the hydraulic gradient dh/dl and the hydraulic conductivity K are held constant, v is independent of the angle θ. This is true even for θ values greater than 90° when the flow is being forced up through the cylinder against gravity.

We have noted that the specific discharge v has the dimensions of a velocity, or flux. For this reason it is sometimes known as the *Darcy velocity* or *Darcy flux*. The specific discharge is a macroscopic concept and it is easily measured. It must be clearly differentiated from the microscopic velocities associated with the actual paths of individual particles of water as they wind their way through the grains of sand (Figure 2.2). The microscopic velocities are real, but they are probably impossible to measure. In the remainder of the chapter we will work exclusively with concepts of flow on a macroscopic scale. Despite its dimensions we will not refer to v as a velocity; rather we will utilize the more correct term, *specific discharge*.

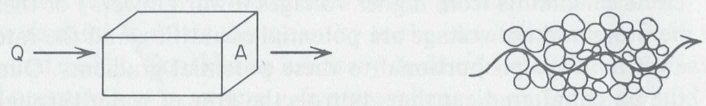

Figure 2.2 Macroscopic and microscopic concepts of groundwater flow.

This last paragraph may appear innocuous, but it announces a decision of fundamental importance. When we decide to analyze groundwater flow with the Darcian approach, it means, in effect, that we are going to replace the actual ensemble of sand grains (or clay particles or rock fragments) that make up the porous medium by a representative continuum for which we can define macroscopic parameters, such as the hydraulic conductivity, and utilize macroscopic laws, such as Darcy's law, to provide macroscopically averaged descriptions of the microscopic behavior. This is a conceptually simple and logical step to take, but it rests on some knotty theoretical foundations. Bear (1972), in his advanced text on porous-media flow, discusses these foundations in detail. In Section 2.12, we will further explore the interrelationships between the microscopic and macroscopic descriptions of groundwater flow.

Darcy's law is an empirical law. It rests only on experimental evidence. Many attempts have been made to derive Darcy's law from more fundamental physical laws, and Bear (1972) also reviews these studies in some detail. The most successful approaches attempt to apply the Navier-Stokes equations, which are widely known in the study of fluid mechanics, to the flow of water through the pore channels of idealized conceptual models of porous media. Hubbert (1956) and Irmay (1958) were apparently the earliest to attempt this exercise.

This text will provide ample evidence of the fundamental importance of Darcy's law in the analysis of groundwater flow, but it is worth noting here that it is equally important in many other applications of porous-media flow. It describes the flow of soil moisture and is used by soil physicists, agricultural engineers, and soil mechanics specialists. It describes the flow of oil and gas in deep geological formations and is used by petroleum reservoir analysts. It is used in the design of filters by chemical engineers and in the design of porous ceramics by materials scientists. It has even been used by bioscientists to describe the flow of bodily fluids across porous membranes in the body.

Darcy's law is a powerful empirical law and its components deserve our more careful attention. The next two sections provide a more detailed look at the physical significance of the hydraulic head h and the hydraulic conductivity K.

2.2 Hydraulic Head and Fluid Potential

The analysis of a physical process that involves flow usually requires the recognition of a potential gradient. For example, it is known that heat flows through solids from higher temperatures toward lower and that electrical current flows through electrical circuits from higher voltages toward lower. For these processes, the temperature and the voltage are potential quantities, and the rates of flow of heat and electricity are proportional to these potential gradients. Our task is to determine the potential gradient that controls the flow of water through porous media.

Fortunately, this question has been carefully considered by Hubbert in his classical treatise on groundwater flow (Hubbert, 1940). In the first part of this section we will review his concepts and derivations.

Hubbert's Analysis of the Fluid Potential

Hubbert (1940) defines *potential* as "a physical quantity, capable of measurement at every point in a flow system, whose properties are such that flow always occurs from regions in which the quantity has higher values to those in which it has lower, regardless of the direction in space" (p. 794). In the Darcy experiment (Figure 2.1) the hydraulic head h, indicated by the water levels in the manometers, would appear to satisfy the definition, but as Hubbert points out, "to adopt it empirically without further investigation would be like reading the length of the mercury column of a thermometer without knowing that temperature was the physical quantity being indicated" (p. 795).

Two obvious possibilities for the potential quantity are elevation and fluid pressure. If the Darcy apparatus (Figure 2.1) were set up with the cylinder vertical ($\theta = 0$), flow would certainly occur down through the cylinder (from high elevation to low) in response to gravity. On the other hand, if the cylinder were placed in a horizontal position ($\theta = 90°$) so that gravity played no role, flow could presumably be induced by increasing the pressure at one end and decreasing it at the other. Individually, neither elevation nor pressure are adequate potentials, but we certainly have reason to expect them to be components of the total potential quantity.

It will come as no surprise to those who have been exposed to potential concepts in elementary physics or fluid mechanics that the best way to search out our quarry is to examine the energy relationships during the flow process. In fact, the classical definition of potential as it is usually presented by mathematicians and physicists is in terms of the work done during the flow process; and the work done in moving a unit mass of fluid between any two points in a flow system is a measure of the energy loss of the unit mass.

Fluid flow through porous media is a mechanical process. The forces driving the fluid forward must overcome the frictional forces set up between the moving